Moment distribution method Workbook

Moment distribution method is a highly effective and widely used technique in structural engineering for analyzing indeterminate beams and frames. Developed by Hardy Cross in the 1930s, this method provides engineers with a systematic approach to determine the moments and forces in statically indeterminate structures. Its significance lies in its iterative process that simplifies complex structures into manageable calculations, making it a cornerstone in the field of structural analysis. Whether designing bridges, buildings, or other load-bearing structures, understanding the moment distribution method is essential for ensuring safety, stability, and efficiency.

Introduction to Moment Distribution Method

The moment distribution method is based on the principles of static and compatibility conditions in structures. It allows engineers to analyze structures that are statically indeterminate?those with more support reactions than necessary for static equilibrium. These structures require additional methods to find internal moments and shears, and the moment distribution method offers a practical solution through a step-by-step iterative process.

The core concept revolves around the idea of distributing unbalanced moments at the joints of the structure until equilibrium is achieved. This process involves calculating fixed-end moments, distributing them to connected members, and iteratively adjusting moments based on the stiffness of the members until the moments converge to a stable solution.

Fundamental Concepts

Stiffness and Flexural Rigidity

Central to the moment distribution method are the concepts of stiffness and flexural rigidity. Flexural rigidity, denoted as EI (where E is the modulus of elasticity and I is the moment of inertia), measures a member's resistance to bending. The stiffness at a joint depends on the flexural rigidity of the connected members and their lengths.

The stiffness (K) for a member at a joint is generally calculated as:

$$ K = \frac{4EI}{L} $$

for internal fixed-end conditions, or based on the boundary conditions for different types of supports.

Fixed-End Moments

When a load is applied to a beam, it induces moments at the ends if the ends are fixed. These moments, known as fixed-end moments (FEM), are calculated based on the type and position of loads. They serve as the starting point in the analysis, representing the moments that would exist if the supports or joints were rigidly fixed.

For example, for a uniformly distributed load w over a span L, the fixed-end moments are:

- At each end: $\pm \frac{wL^2}{12}$

These fixed-end moments are then adjusted during the iterative process to account for the actual support conditions.

Step-by-Step Process of the Moment Distribution Method

The method involves several key steps, which are repeated iteratively until the moments stabilize. The main steps are:

1. Calculate Fixed-End Moments

Begin by computing the fixed-end moments for all members subjected to loads, based on their boundary conditions and load types.

2. Assign Stiffness and Distribute Moments

Determine the stiffness of each member at the joints. Using these stiffness values, distribute the fixed-end moments to the adjacent members proportionally, based on their relative stiffness.

Distribution Factor (DF):

$$ DF_{ij} = \frac{K_{ij}}{\sum K_{i}} $$

where K_{ij} is the stiffness of member ij at joint i.

The fixed-end moments are then distributed to the connected members according to these factors.

3. Carry Over Moments

After distributing the moments, the moments at the joints are not yet in equilibrium. Therefore, the moments are carried over to the other ends of the members using a carry-over factor, typically half of the distributed moment for uniformly distributed loads.

Carry-Over Factor (COF):

$$COF = 0.5$$

for uniform or simple loads, but may vary depending on load cases.

4. Repeat Until Convergence

The process of distributing moments and carrying them over is repeated, adjusting the moments at each joint. With each iteration, the unbalanced moments decrease, approaching an equilibrium state. When the moments change negligibly between successive iterations, the solution is considered converged.

Advantages and Limitations

Advantages of the Moment Distribution Method

- **Systematic and straightforward:** The method follows a clear, step-by-step procedure suitable for manual calculations.
- **Applicable to indeterminate structures:** It effectively handles structures with multiple support reactions.
- **Provides detailed internal moment distribution:** Useful for designing reinforcement and assessing structural behavior.
- **Educational value:** Enhances understanding of structural behavior and load distribution.

Limitations of the Moment Distribution Method

- **Complex for very large structures:** Manual calculations become cumbersome with many members and joints.
- **Assumes linear elastic behavior:** Not suitable for non-linear or plastic analysis.
- **Limited to static analysis:** Dynamic effects require other methods.
- **Requires careful calculation of stiffness and fixed-end moments:** Errors in these initial steps can propagate through the analysis.

Applications in Structural Engineering

The moment distribution method is extensively used in various structural engineering applications, including:

Analysis of Continuous Beams

For beams supported on multiple supports and subjected to loads, the method helps determine the moments at supports and mid-span, essential for designing reinforcement.

Frame Analysis

In multi-story frames, the method assists in calculating moments at joints, aiding in the design of beams and columns.

Bridge Structures

Designing continuous bridges with multiple spans involves complex load distribution, where the moment distribution method simplifies the analysis process.

Reinforced Concrete and Steel Structures

Accurate moment calculations inform reinforcement detailing and ensure the structure's safety and serviceability.

Comparison with Other Structural Analysis Methods

While the moment distribution method is popular for manual calculations, it is often compared with other analysis techniques:

Matrix Methods

- Use stiffness matrices and linear algebra for analysis.
- Suitable for computer-based analysis of large structures.
- More efficient for complex and large systems but less intuitive.

Flexibility Method

- Uses compatibility conditions to solve indeterminate structures.
- Generally more algebraically intensive but can handle a broader range of problems.

Finite Element Method (FEM)

- Numerical technique capable of analyzing highly complex structures.
- Suitable for detailed analysis, including non-linear behavior and dynamic effects.

Despite advances in computational methods, the moment distribution method remains valuable for educational purposes, conceptual understanding, and quick manual calculations.

Practical Tips for Using the Moment Distribution Method

- Always verify fixed-end moments and stiffness calculations.
- Use consistent units throughout the analysis.
- Keep track of moments at each iteration carefully to avoid errors.
- Recognize the limits of the method and consider computer-aided analysis for complex structures.
- Use software tools for large or intricate structures to save time and improve accuracy.

Conclusion

The moment distribution method is a fundamental technique in the arsenal of structural engineers, providing a practical and insightful way to analyze indeterminate structures. Its systematic approach, rooted in engineering principles of stiffness, load distribution, and equilibrium, enables accurate determination of internal moments critical for safe and efficient design. While modern computational tools have supplemented manual methods, understanding the moment distribution method remains essential for grasping the behavior of complex structures and for educational purposes. With its blend of simplicity and effectiveness, it continues to be a valuable technique in structural analysis, ensuring that structures are both resilient and economical.

Understanding the Moment Distribution Method: A Comprehensive Guide

In the realm of structural engineering, particularly in the analysis of statically indeterminate beams and frames, the Moment Distribution Method stands out as a fundamental and intuitive technique. This method, developed by Hardy Cross in the 1930s, offers engineers a systematic approach to determine the moments and reactions in complex structures through a process of iterative balancing. Whether you're a student aiming to grasp the basics or a practicing engineer refining your analytical toolkit, understanding the moment distribution method is essential for solving indeterminate structures efficiently and accurately.

What is the Moment Distribution Method?

The moment distribution method is a stiffness method used to analyze continuous beams and frames that are statically indeterminate. Unlike simple determinate structures, indeterminate structures have more support reactions than necessary for equilibrium, making their analysis more complex. The moment distribution method simplifies this complexity by distributing moments gradually until the structure reaches equilibrium.

At its core, the method involves:

- Calculating fixed-end moments based on loadings.
- Distributing these moments to adjacent members according to their stiffness.
- Applying correction steps iteratively to balance moments at the supports.
- Achieving convergence when moments stabilize within a specified tolerance.

Key Concepts and Terminology

Before diving into the step-by-step process, it?s crucial to familiarize yourself with some fundamental concepts:

- Stiffness of a member: The resistance of a member to bending, often represented as a stiffness coefficient in the method.
- Distribution factor: The proportion of unbalanced moment to be distributed to a joint, determined by the relative stiffnesses of the connected members.
- Carry-over factor: The proportion of moment transferred from one end of a member to the other, typically 0.5 for symmetrical members.
- Fixed-end moments (FEM): The moments that would occur at the ends of a member if it were fixed at both ends under a particular load.

Step-by-Step Procedure of the Moment Distribution Method

- Identify the Structure and Supports

Begin by clearly sketching the structure, marking all supports, spans, loads, and boundary conditions. For each joint, note the number of connected members.

- Calculate Fixed-End Moments (FEM)

Using the loadings (uniform, point loads, etc.), compute the fixed-end moments for each member as if their ends were fixed. Standard formulas or tables are often used for common load types:

- For a uniformly distributed load w over a span L:

- Fixed-end moment at each end:

$$
M_{fixed} = -\frac{wL^2}{12}
$$

- For point loads and other cases, refer to standard FEM formulas.

- Determine the Distribution Factors (DF)

For each joint, calculate the distribution factor for each connected member:

$$
DF = \frac{K}{\sum K}
$$

where K is the stiffness of the member at that joint.

- For a simply supported beam segment:

$$
K = \frac{4EI}{L}
$$

- For a continuous beam or frame, the stiffness considers the member's properties and boundary conditions.

- Initial Moment Distribution

- At each joint with unbalanced moments, distribute the fixed-end moments to adjacent supports according to their distribution factors.
- The unbalanced moment at each joint is initially the FEM at that joint.

- Carry-Over of Moments

- After each distribution step, transfer a portion of the moment from one end of a member to the other.
- The carry-over factor (usually 0.5 for symmetrical members) determines how much of the distributed moment is carried over:

$$
\text{Carry-over moment} = \text{Distributed moment} \times \text{Carry-over factor}
$$

- For each member, add the carry-over moments to the opposite ends.

- Iterative Balancing

- Repeat the distribution and carry-over steps:
- Distribute the residual moments to the supports.
- Carry over the moments along the members.
- Continue until the moments at all joints converge within a pre-defined small tolerance (e.g., 1e-4).

- Calculate Final Member Moments and Reactions

- Once the iterative process stabilizes, sum all moments at each support and along each member.

- Use the moments to compute reactions, shear forces, and deflections as needed.

Practical Example: Continuous Beam with Uniform Load

Imagine a simple continuous beam spanning three supports (A, B, and C) with a uniform load w. Here's a simplified outline:

- Calculate FEM at each end:

- For spans AB, BC, etc., based on load and span length.

- Compute distribution factors at each joint:

- For joint B, connect to spans AB and BC, compute DF_{AB} and DF_{BC}.

- Distribute initial fixed-end moments:

- Allocate FEMs to adjacent supports according to DFs.

- Carry over moments:

- Transfer moments along each member with a factor of 0.5.

- Iterate:

- Continue distributing and carrying over until moments stabilize.

- Determine support reactions:

- Summate moments at supports and apply equilibrium equations.

This process reveals the moments at supports and the internal moments within the members, providing a comprehensive picture of the structure's response.

Advantages and Limitations

Advantages:

- Intuitive and straightforward, especially for continuous beams and frames.
- Suitable for hand calculations of small to medium-sized structures.
- Offers insight into the distribution of moments within the structure.

Limitations:

- Becomes cumbersome for very large or complex structures without computational aid.
- Less efficient than matrix methods for very high degrees of indeterminacy.
- Assumes linear elastic behavior and small deflections.

Modern Applications and Software

While the moment distribution method is invaluable for educational purposes and small structures, modern structural analysis software (like SAP2000, ETABS, or STAAD.Pro) employs matrix methods that can handle larger, more complex structures more efficiently. Nonetheless, understanding the moment distribution technique provides valuable insight into the behavior of indeterminate structures and helps validate numerical results.

Final Thoughts

The Moment Distribution Method remains a foundational analysis technique in structural engineering. Its iterative nature fosters a deep understanding of how moments are redistributed in indeterminate structures, promoting a more intuitive grasp of structural behavior. Mastery of this method not only aids in manual calculations but also enhances overall structural intuition, making it an essential part of any structural engineer?s analytical skill set. Whether used as a teaching tool or a quick analysis technique, the moment distribution method continues to be a testament to engineering ingenuity and simplicity.

QuestionAnswer What is the moment distribution method in structural analysis? The moment distribution method is an iterative technique used to analyze statically indeterminate structures by distributing and balancing moments at joints until equilibrium is achieved. How does the moment distribution method simplify the analysis of indeterminate frames? It simplifies analysis by breaking down complex structures into a series of joint calculations, distributing moments based on stiffness, and iteratively balancing moments without solving simultaneous equations directly. What are the main steps involved in performing the moment distribution method? The main steps include fixing moments at joints, calculating carry-over and distribution factors, distributing moments to adjacent members, and repeating the process until the moments converge to a stable solution. What are the advantages of using the moment distribution method? Advantages include its systematic approach, suitability for hand calculations, and ease of understanding, especially for analyzing continuous beams and frames with multiple supports. Are there limitations to the moment distribution method? Yes, it can become cumbersome for very large structures with numerous members, and it is less efficient than matrix methods for complex, computer-aided analysis. How is the moment distribution method relevant in modern structural engineering practice? While more advanced computer-based methods are now common, the moment distribution method remains a valuable educational tool and is useful for quick, approximate analysis of indeterminate structures.

Related keywords: structural analysis, indeterminate structures, stiffness method, load distribution, beam analysis, continuous beams, structural flexibility, moment calculations, stiffness matrix, static indeterminacy

Regula falsi method advantages and disadvantages

Regula Falsi Method Advantages and Disadvantages

The regula falsi method, also known as the false position method, is a numerical technique used to find approximate solutions to equations where the root is unknown. It is a bracketing method that combines the features of the bisection method and the secant method, aiming to improve convergence speed while maintaining reliability. Like any numerical approach, it offers a set of advantages that make it appealing for certain applications, but it also exhibits notable disadvantages that can limit its effectiveness in specific scenarios. Understanding these benefits and drawbacks is crucial for practitioners to decide when and how to employ the regula falsi method effectively.

Advantages of the Regula Falsi Method

1. Guaranteed Convergence Under Certain Conditions

One of the primary advantages of the regula falsi method is its guaranteed convergence, provided that the initial interval brackets a root and the function is continuous. Since the method always maintains an interval where the function changes sign, it ensures that the root lies within the bounds during each iteration. This reliability makes it a safe choice compared to open methods like the secant method, which may not always converge.

2. Simplicity of Implementation

The regula falsi method is straightforward to implement computationally. Its core involves calculating a linear approximation between two points and updating the interval based on the sign of the function at the approximated root. The simplicity of the algorithm makes it accessible even for beginners and easy to incorporate into larger computational routines or software.

3. Faster Convergence Than the Bisection Method

Compared to the bisection method, which halves the interval in each iteration, regula falsi often converges more quickly because it uses a secant-like approach to estimate the root. By adjusting the interval based on a linear approximation, it tends to reduce the number of iterations needed, especially when the initial interval is well-chosen and the function behaves approximately linearly near the root.

4. Fewer Function Evaluations Than Bisection

While both methods require function evaluations at each step, regula falsi often achieves a desired accuracy with fewer evaluations compared to bisection. This efficiency can be particularly valuable when function evaluations are computationally expensive or time-consuming.

5. Flexibility With Different Types of Functions

The method is applicable to a wide range of functions, including nonlinear and complex functions, as long as the initial interval brackets a root. Its reliance on linear approximation rather than derivatives makes it suitable even when derivative information is unavailable or difficult to compute.

Disadvantages of the Regula Falsi Method

1. Slow or Stagnant Convergence in Certain Cases

Despite its faster convergence relative to bisection, regula falsi can experience slow progress or stagnation, especially when the function exhibits certain behaviors. For example, if one endpoint of the interval is close to the root and the other is far, the method may repeatedly refine the approximation near one side without significantly improving the estimate overall. This phenomenon is known as "stagnation" and can lead to an impractical number of iterations.

2. Sensitivity to the Initial Interval

The effectiveness of the regula falsi method heavily depends on the choice of the initial bracketing interval. If the initial interval does not contain a root or is poorly chosen, the method may fail to converge or converge very slowly. The method requires a good initial approximation to be efficient and reliable.

3. Potential for Non-Progressive Iterations

In some cases, the method can get "stuck" or make negligible improvements over iterations. This occurs when the linear approximation becomes poor due to the nonlinear nature of the function, especially near inflection points or discontinuities. As a result, the method might oscillate or hover without substantial progress towards the root.

4. Not as Fast as Higher-Order Methods

Compared to methods like Newton-Raphson or secant methods, regula falsi generally converges more slowly because it is a bracketing method with linear convergence. For functions where derivative information is available and the function behaves well, higher-order methods can achieve quadratic or superlinear convergence, making regula falsi less efficient.

5. Computational Overhead in Some Variants

While the basic regula falsi method is simple, some advanced variants attempt to improve convergence by modifying how the new approximation is computed. These variants can introduce additional computational steps or complexity, which may negate some of the simplicity advantages of the original method.

Summary of Advantages and Disadvantages

Summary of Advantages

- Guaranteed convergence when the initial interval brackets a root
- Simple to implement and understand
- Generally faster than the bisection method
- Requires fewer function evaluations compared to bisection in many cases
- Applicable to a wide range of functions without needing derivatives

Summary of Disadvantages

- Can experience slow convergence or stagnation in certain functions
- Highly dependent on the choice of initial interval
- May get stuck or oscillate without significant progress
- Slower convergence compared to higher-order methods like Newton-Raphson
- Potential additional computational complexity in advanced variants

Conclusion

The regula falsi method remains a valuable tool in the numerical analyst's toolkit, especially when robustness and guaranteed convergence are prioritized over speed. Its advantages?such as simplicity, reliability, and efficiency?make it suitable for a variety of problems, particularly when derivative information is unavailable or the function is complex. However, its disadvantages, including potential stagnation and sensitivity to initial conditions, mean that practitioners should exercise caution and consider alternative methods if rapid convergence is necessary or if the function exhibits challenging behavior. Understanding the balance between these advantages and disadvantages allows for more informed method selection and more effective problem-solving in numerical root-finding tasks.

Regula Falsi Method Advantages and Disadvantages

The regula falsi method, also known as the false position method, is a popular numerical technique used for finding roots of continuous functions. This iterative method provides an efficient way to approximate solutions to equations where analytical methods may be cumbersome or impossible. Its simplicity, combined with its ability to converge quickly under certain conditions, makes it a preferred choice among mathematicians, engineers, and scientists. However, like any numerical technique, the regula falsi method also has its limitations and drawbacks that are essential to understand for effective application.

Understanding the Regula Falsi Method

Before diving into its advantages and disadvantages, it?s important to understand the basic concept of the regula falsi method.

Basic Concept

The regula falsi method is based on the idea of linear interpolation. Given a continuous function $f(x)$ and two points a and b such that $f(a)$ and $f(b)$ have opposite signs (indicating a root lies between them), the method approximates the root by drawing a straight line connecting the points $(a, f(a))$ and $(b, f(b))$. The intersection of this line with the x-axis provides a new approximation of the root, which then replaces either a or b based on the sign of the function at this intersection.

Step-by-Step Process

- Choose initial points a and b such that $f(a) \times f(b)$
- Calculate the point c where the straight line connecting $(a, f(a))$ and $(b, f(b))$ intersects the x-axis:

$$
c = b - \frac{f(b)(a - b)}{f(a) - f(b)}
$$

- Evaluate $f(c)$. If $f(c)$ is sufficiently close to zero or within a desired tolerance, c is taken as the root.
- Otherwise, replace either a or b with c, depending on the sign of $f(c)$, and repeat the process.

Advantages of the Regula Falsi Method

Despite being a relatively simple numerical root-finding technique, the regula falsi method offers several notable advantages that make it appealing for various applications.

1. Simplicity and Ease of Implementation

- The regula falsi method relies on straightforward calculations involving linear interpolation, making it easy to implement even for beginners.
- No complex mathematical concepts or advanced programming skills are necessary, which allows for quick coding and testing.

2. Guaranteed Convergence under Certain Conditions

- When the initial interval $[a, b]$ contains a root and f is continuous, the method guarantees convergence to a root, provided the initial guesses are chosen appropriately.
- Unlike some methods, such as the secant method, the regula falsi method always converges if the initial bracketing is correct.

3. Faster than Bisection in Many Cases

- Since regula falsi uses linear interpolation, it often converges more rapidly than the bisection method, which simply bisects the interval regardless of the function's behavior.
- Especially when the function is well-behaved near the root, the method can achieve desired accuracy in fewer iterations.

4. No Need for Derivatives

- Unlike Newton-Raphson, the regula falsi method does not require the computation of derivatives, making it suitable for functions where derivatives are difficult or expensive to evaluate.

5. Suitable for Functions with Multiple Roots

- The method can be adapted to find multiple roots within a given interval by applying it iteratively over different subintervals.

Disadvantages of the Regula Falsi Method

While the regula falsi method has its merits, it also suffers from several significant drawbacks that can limit its effectiveness in certain scenarios.

1. Slow or Non-Uniform Convergence in Some Cases

- The method can experience "stalling" or very slow convergence when one endpoint remains fixed during iterations, especially if the function behaves poorly near the root.
- This occurs when the new approximation continually replaces only one endpoint, leading to a linear convergence rate similar to the bisection method rather than quadratic.

2. Dependence on Good Initial Brackets

- The success and efficiency of the method heavily depend on choosing initial points \(a\) and \(b\) such that \(f(a)\) and \(f(b)\) have opposite signs.
- Poor choices can lead to divergence, stagnation, or convergence to an incorrect root.

3. Potential for Stagnation

- In some cases, particularly with functions that are nearly flat or have multiple roots close to each

other, the method may "stall," where the approximations do not improve significantly over iterations.
- This stagnation is problematic when quick convergence is desired.

4. Limited to Continuous Functions with Sign Changes

- The regula falsi method requires a continuous function with a known sign change over the interval. It is not suitable for functions that are discontinuous or do not satisfy this condition.
- Additional preliminary analysis is often necessary before applying the method.

5. Numerical Instability and Precision Issues

- When \(f(a) \approx f(b)\), the denominator in the interpolation formula becomes very small, leading to potential numerical instability.
- This can cause inaccuracies or divergence in the root approximation, especially when working with floating-point arithmetic.

Features and Variants

To address some of the shortcomings of the basic regula falsi method, several variants have been developed:

- Illinois Method: Adjusts the weight of the fixed endpoint to improve convergence.
- Pegasus Method: Uses a modified interpolation formula for faster convergence.
- Modified Regula Falsi: Incorporates dynamic updates to endpoints to prevent stagnation.

Each of these variants aims to retain the simplicity of the original method while improving convergence speed and stability.

Practical Applications and Suitability

The regula falsi method is particularly useful in scenarios where:

- The function is continuous and the initial bracket is known.
- Derivative information is unavailable or expensive to compute.
- Quick implementation and results are required.

However, for functions with complex behavior, multiple roots, or where high precision is crucial, other methods like Newton-Raphson or secant might be more appropriate.

Conclusion

The regula falsi method remains a fundamental numerical technique for root-finding due to its simplicity and guaranteed convergence under the right conditions. Its strengths lie in its straightforward implementation, no requirement for derivatives, and often faster convergence than bisection, especially for well-behaved functions. Nevertheless, it faces notable challenges, such as potential slow convergence, stagnation issues, and sensitivity to the initial interval selection.

For practitioners, understanding both the advantages and limitations of the regula falsi method is vital for its effective application. When combined with strategic initial guesses and possible variants, it can serve as a powerful tool in the numerical analyst?s toolkit. However, for more complex functions or demanding precision, alternative methods or hybrid approaches may offer better performance. Overall, the regula falsi method exemplifies the balance between simplicity and effectiveness in numerical root-finding techniques.

QuestionAnswer What are the main advantages of the regula falsi method? The regula falsi method is simple to implement, converges more reliably than some other methods for certain functions, and guarantees a root within the initial interval as long as the function is continuous and the initial guesses bracket the root. What are the disadvantages of using the regula falsi method? One major disadvantage is that it can converge slowly, especially if the function is flat near the root, and it may get stuck with one endpoint not moving if the function's values at the interval ends do not change significantly. How does the regula falsi method compare to the bisection method in terms of efficiency? While the regula falsi method often converges faster than the bisection method due to its use of linear interpolation, it can sometimes suffer from slow convergence or stagnation, unlike the guaranteed steady convergence of bisection. Can the regula falsi method be used for all types of functions? The method works best for continuous functions where the initial interval brackets a root; however, for functions with multiple roots or discontinuities, its effectiveness may be limited or require modifications. What are some improvements or variants of the regula falsi method to overcome its disadvantages? Variants like the Illinois and Anderson methods introduce modifications to the regula falsi technique to accelerate convergence and prevent stagnation, making the root-finding process more efficient. Is the regula falsi method suitable for automated or iterative root-finding algorithms? Yes, it is suitable for automated algorithms due to its straightforward implementation, but care must be taken to monitor convergence speed and avoid stagnation, especially for challenging functions.

Related keywords: regula falsi method, false position method, numerical analysis, root finding, bracketing methods, convergence rate, advantages, disadvantages, iterative methods, computational efficiency

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